Devre Analizinde Laplace Dönüşümü

Laplace dönüşümü kullanılarak devre analizi 3 adımda yapılır:

- Devre bileşenleri zaman uzayından s uzayına çevrilir.
- Devre analiz yöntemleri kullanılarak devre çözümlenir.
- Ters Laplace dönüşümü yapılarak çözüm s uzayından zaman uzayına çevrilir.

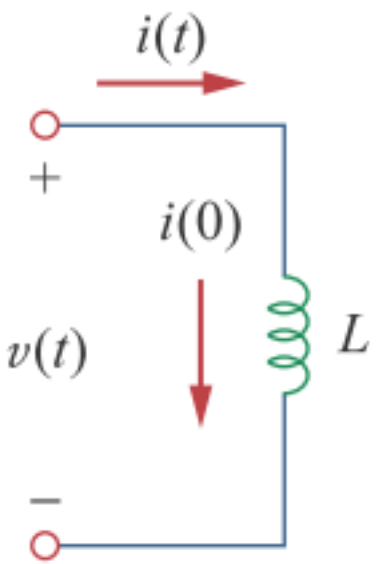
Devre Analizinde Laplace Dönüşümü

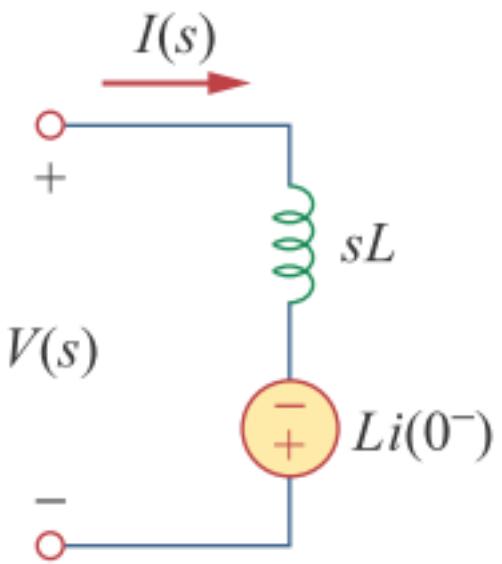
Direnç $v(t) = Ri(t) \qquad V(s) = RI(s)$

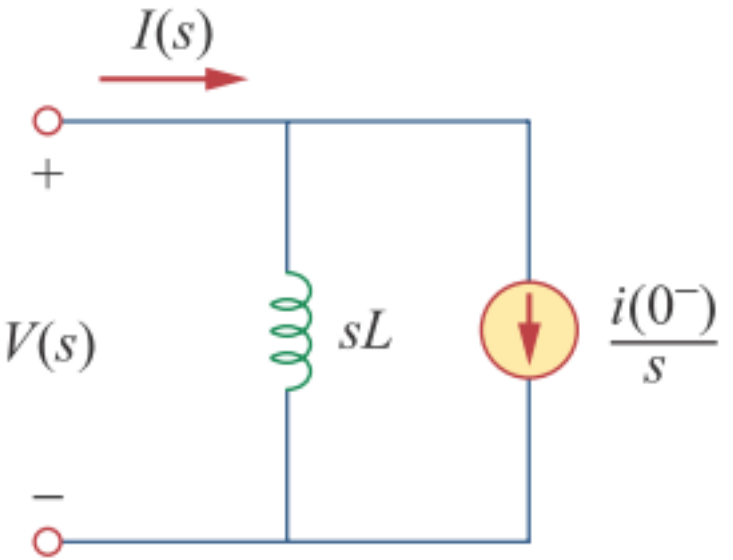
Bobin $v(t) = L\dfrac{di(t)}{dt}$

$$V(s) = L[sI(s) - i(0^-)] = sLI(s) - Li(0^-)$$

$$I(s) = \frac{1}{sL}V(s) + \frac{i(0^-)}{s}$$







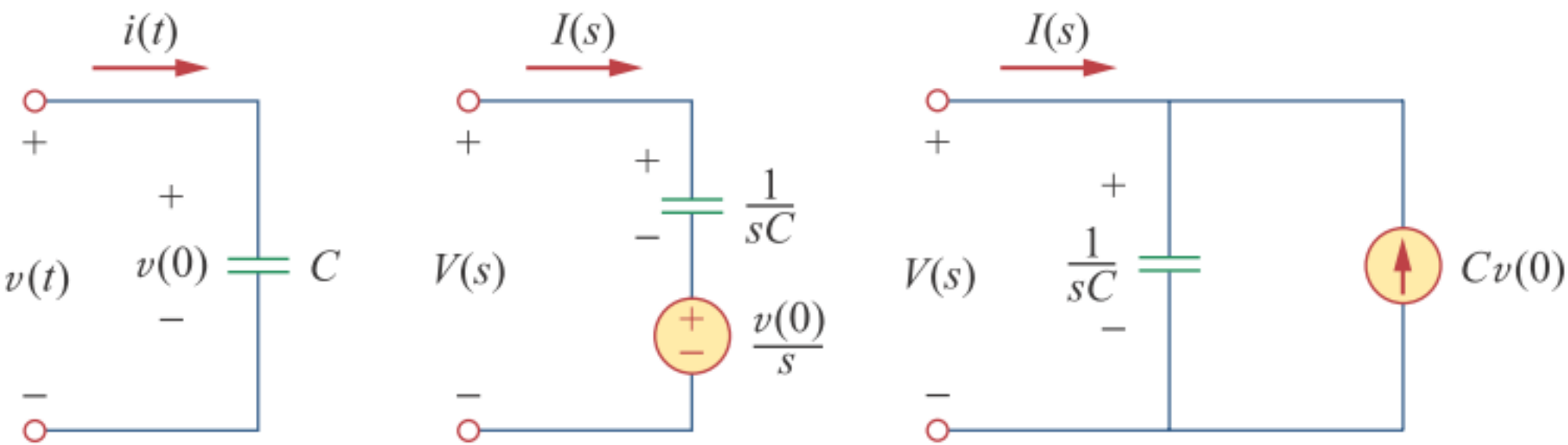
Devre Analizinde Laplace Dönüşümü

Kapasitör

$$i(t) = C\frac{dv(t)}{dt}$$

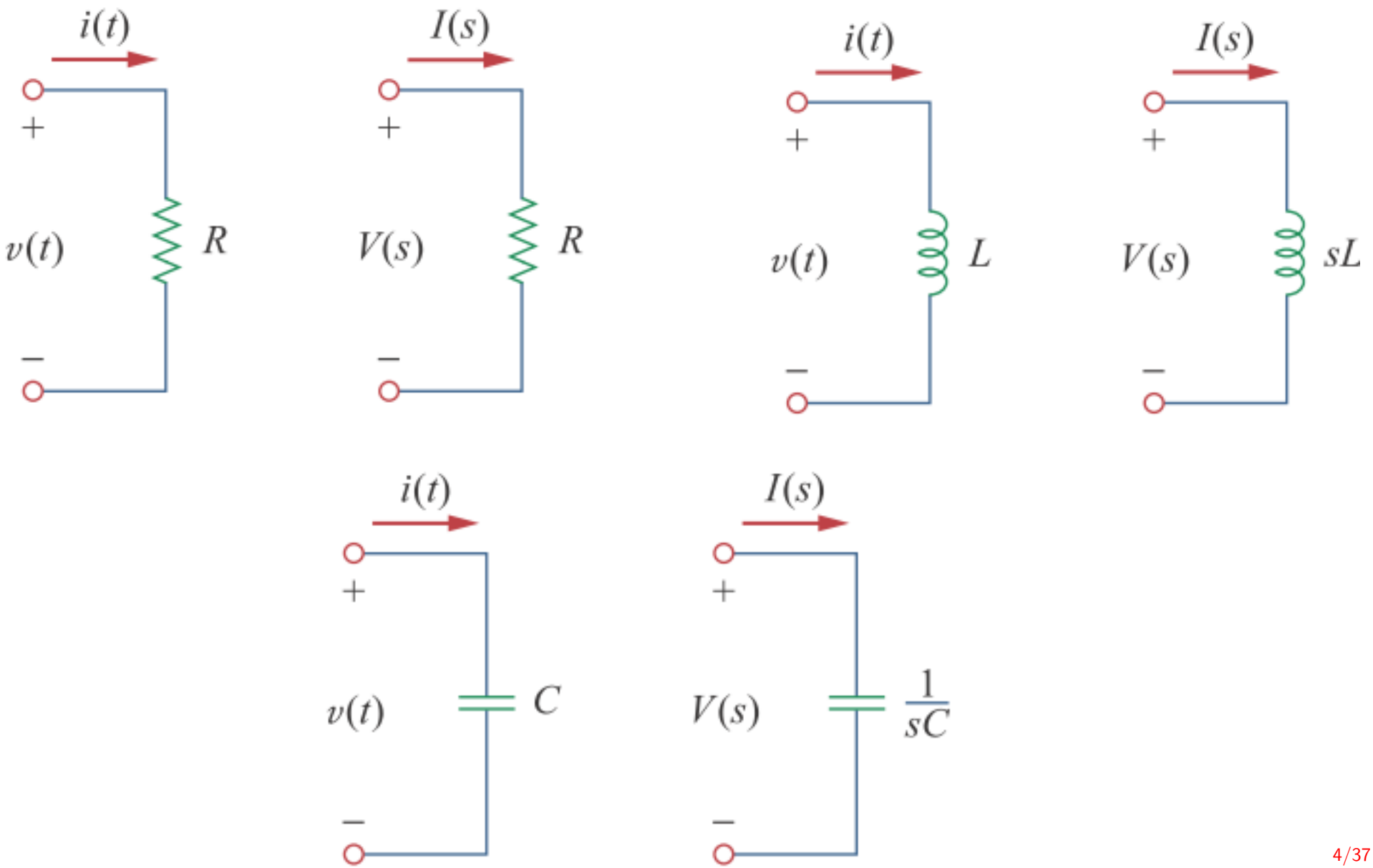
$$I(s) = C[sV(s) - v(0^-)] = sCV(s) - Cv(0^-)$$

$$V(s) = \frac{1}{sC}I(s) + \frac{v(0^-)}{s}$$



Devre Analizinde Laplace Dönüşümü

Başlangıç durumları 0 ise:



Devre Analizinde Laplace Dönüşümü

Başlangıç durumları 0 ise:

$$\text{Resistor:} \quad V(s) = RI(s)$$

$$\text{Inductor:} \quad V(s) = sLI(s)$$

$$\text{Capacitor:} \quad V(s) = \frac{1}{sC} I(s)$$

$$Z(s) = \frac{V(s)}{I(s)}$$

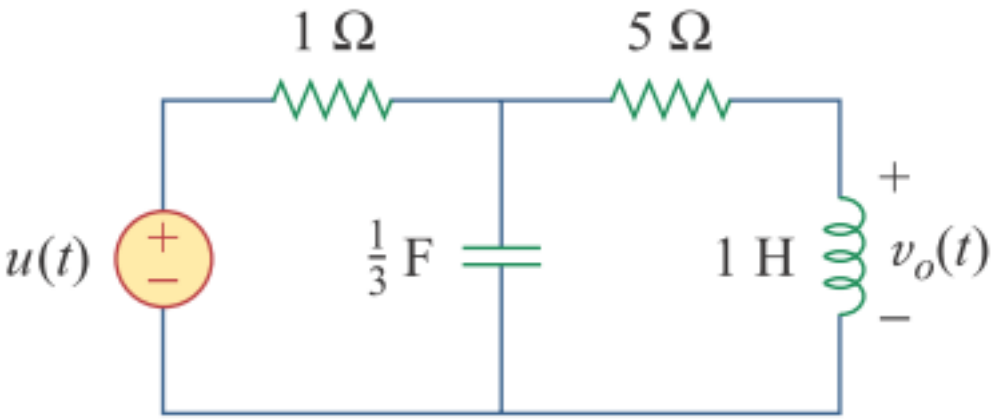
$$\text{Resistor:} \quad Z(s) = R$$

$$\text{Inductor:} \quad Z(s) = sL$$

$$\text{Capacitor:} \quad Z(s) = \frac{1}{sC}$$

Devre Analizinde Laplace Dönüşümü

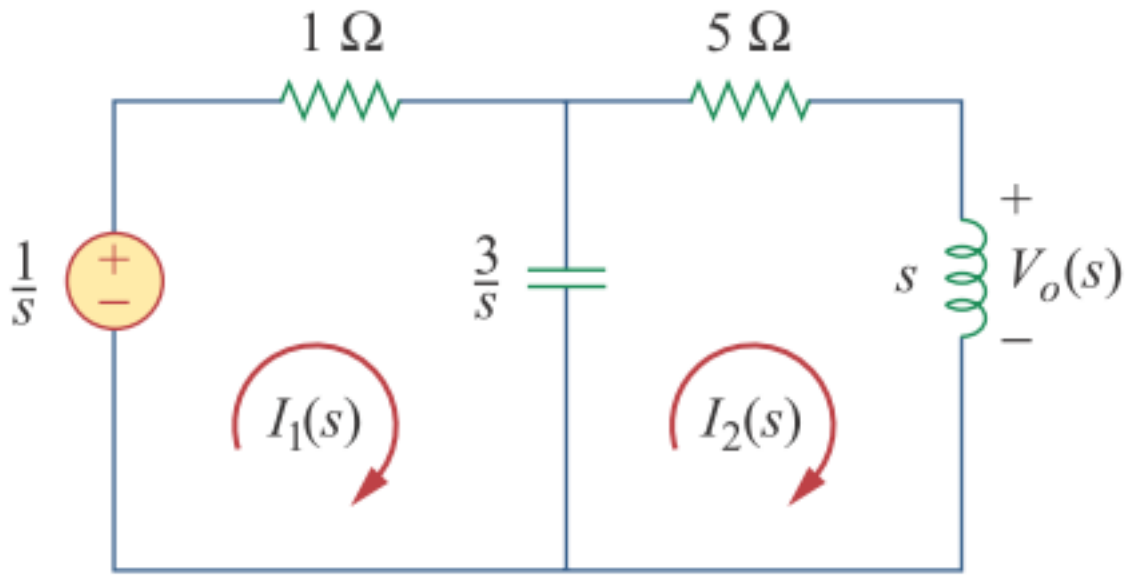
Soru: Devrede başlangıç koşulları 0 ise $v_o(t)$'yi bulunuz.



$$u(t) \quad \Rightarrow \quad \frac{1}{s}$$

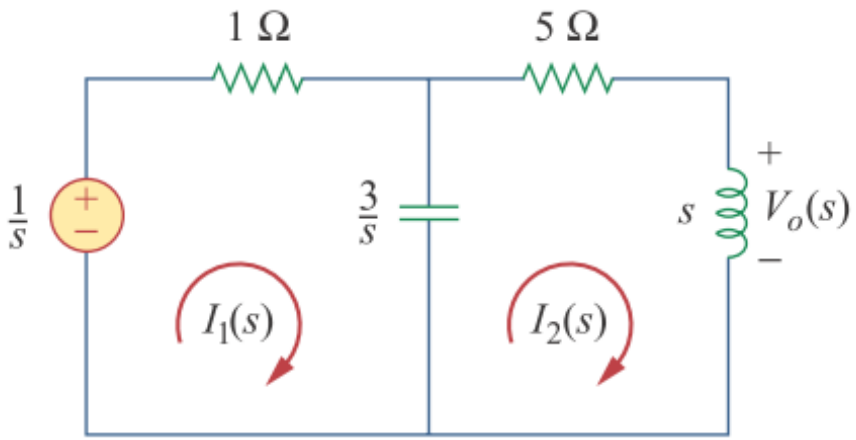
$$1\ \text{H} \quad \Rightarrow \quad sL = s$$

$$\frac{1}{3}\text{F} \quad \Rightarrow \quad \frac{1}{sC} = \frac{3}{s}$$



Devre Analizinde Laplace Dönüşümü

Devrede başlangıç koşulları 0 ise $v_o(t)$'yi bulunuz.



$$\frac{1}{s} = \left(1 + \frac{3}{s}\right)I_1 - \frac{3}{s}I_2$$

$$0 = -\frac{3}{s}I_1 + \left(s + 5 + \frac{3}{s}\right)I_2$$

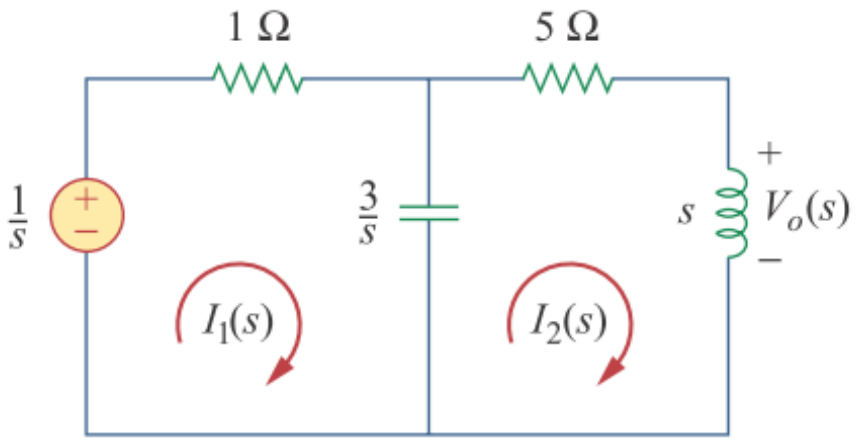
$$I_1 = \frac{1}{3}(s^2 + 5s + 3)I_2$$

$$\frac{1}{s} = \left(1 + \frac{3}{s}\right)\frac{1}{3}(s^2 + 5s + 3)I_2 - \frac{3}{s}I_2$$ $3s$ ile çarpalım

$$3 = (s^3 + 8s^2 + 18s)I_2 \qquad I_2 = \frac{3}{s^3 + 8s^2 + 18s}$$

Devre Analizinde Laplace Dönüşümü

Devrede başlangıç koşulları 0 ise $v_o(t)$'yi bulunuz.



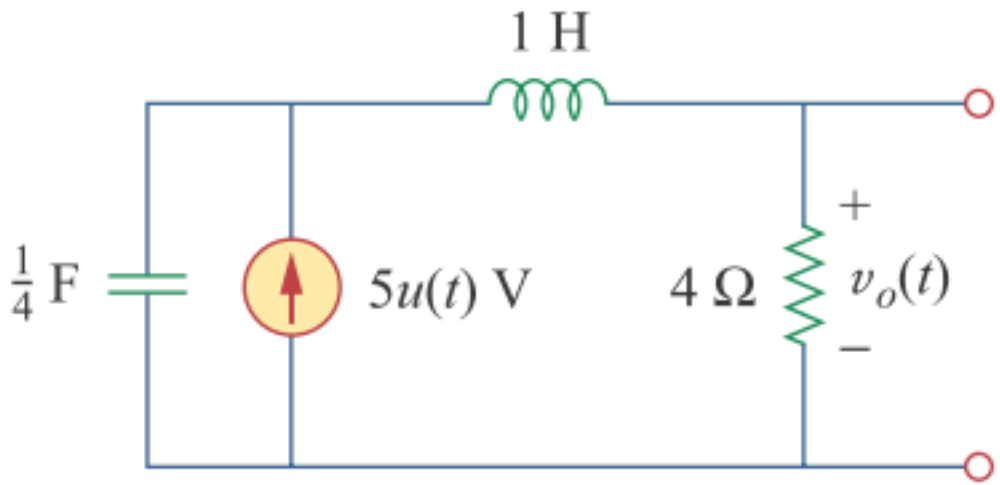
$$I_2 = \frac{3}{s^3 + 8s^2 + 18s}$$

$$V_o(s) = sI_2 = \frac{3}{s^2 + 8s + 18} = \frac{3}{\sqrt{2}} \frac{\sqrt{2}}{(s+4)^2 + (\sqrt{2})^2}$$

$$v_o(t) = \frac{3}{\sqrt{2}} e^{-4t} \sin \sqrt{2}t \text{ V}, \qquad t \geq 0$$

Devre Analizinde Laplace Dönüşümü

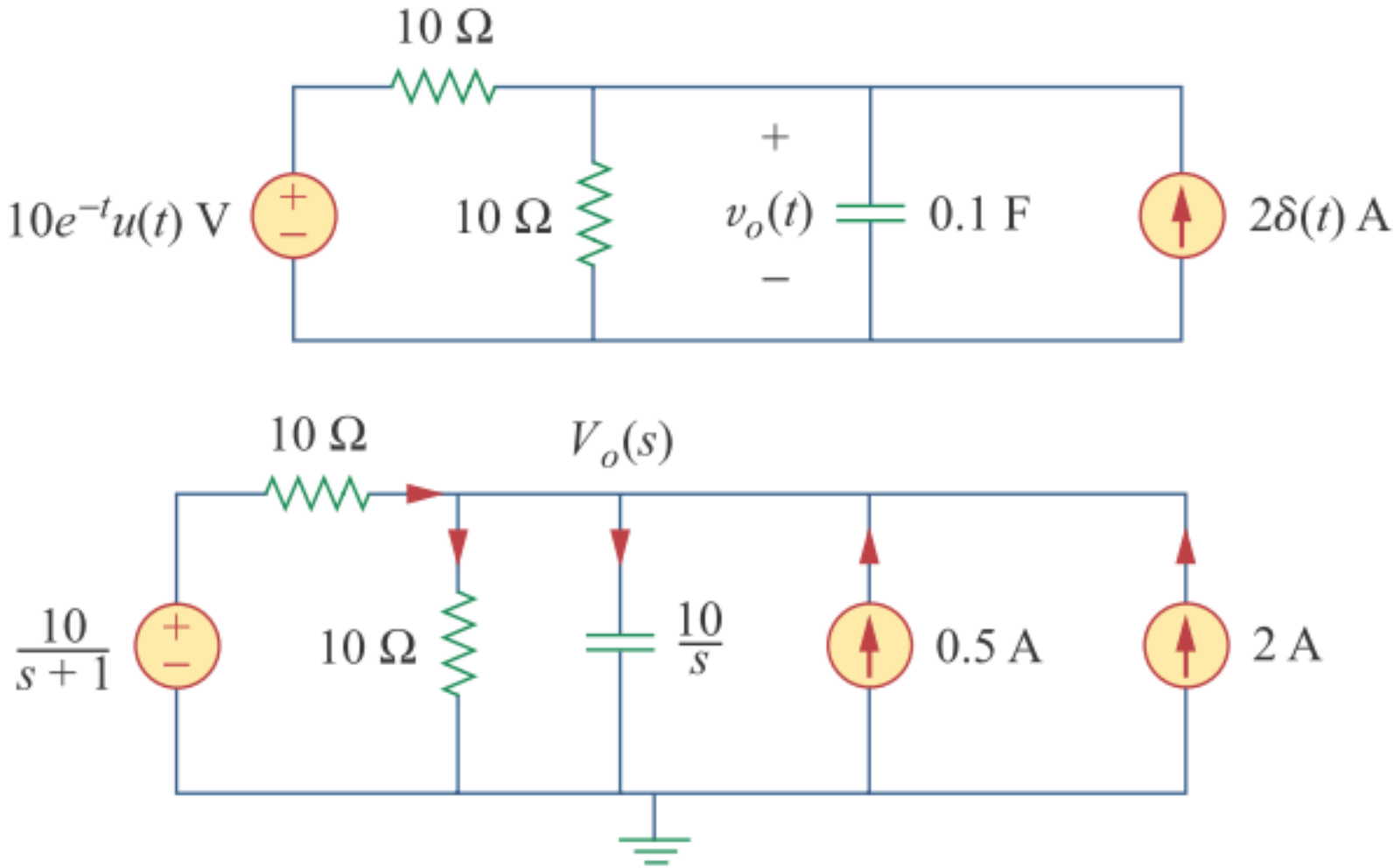
Ödev: Devrede başlangıç koşulları 0 ise $v_o(t)$'yi bulunuz.



$$20(1 - e^{-2t} - 2te^{-2t})u(t) \text{ V}.$$

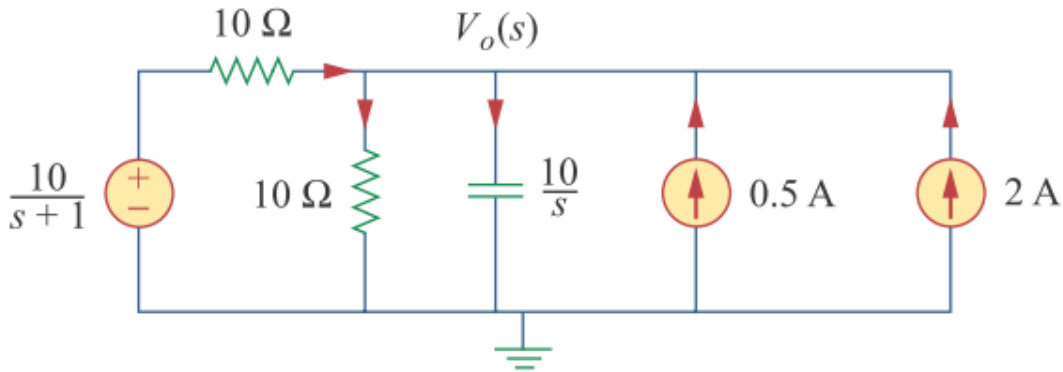
Devre Analizinde Laplace Dönüşümü

Soru: $v_o(0) = 5V$ ise $v_o(t)$'yi bulunuz.



Devre Analizinde Laplace Dönüşümü

Soru: $v_o(0) = 5V$ ise $v_o(t)$'yi bulunuz.



$$\frac{10/(s+1) - V_o}{10} + 2 + 0.5 = \frac{V_o}{10} + \frac{V_o}{10/s}$$

$$\frac{1}{s+1} + 2.5 = \frac{2V_o}{10} + \frac{sV_o}{10} = \frac{1}{10}V_o(s+2)$$

$$\frac{10}{s+1} + 25 = V_o(s+2)$$

Devre Analizinde Laplace Dönüşümü

$v_o(0) = 5V$ ise $v_o(t)$'yi bulunuz.

$$\frac{10}{s+1} + 25 = V_o(s+2)$$

$$V_o = \frac{25s + 35}{(s+1)(s+2)} = \frac{A}{s+1} + \frac{B}{s+2}$$

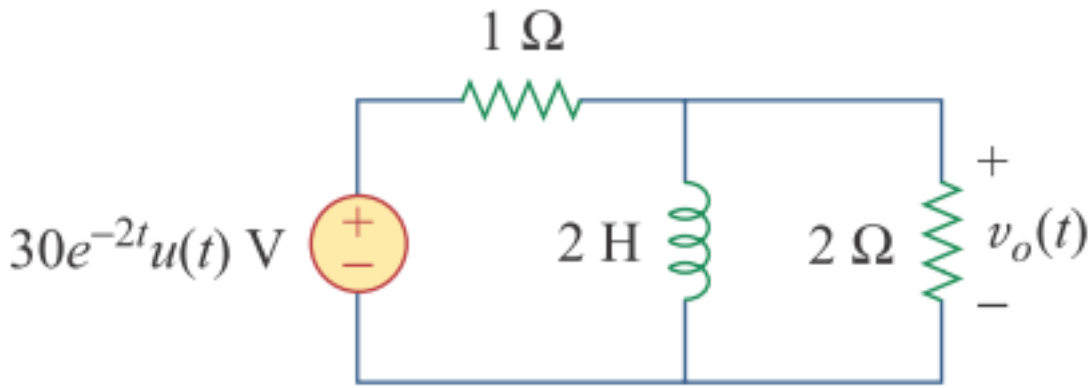
$$A = (s+1)V_o(s)\,|_{s=-1} = \left.\frac{25s+35}{(s+2)}\right|_{s=-1} = \frac{10}{1} = 10$$

$$B = (s+2)V_o(s)\,|_{s=-2} = \left.\frac{25s+35}{(s+1)}\right|_{s=-2} = \frac{-15}{-1} = 15$$

$$V_o(s) = \frac{10}{s+1} + \frac{15}{s+2} \qquad v_o(t) = (10e^{-t} + 15e^{-2t})u(t)\ \text{V}$$

Devre Analizinde Laplace Dönüşümü

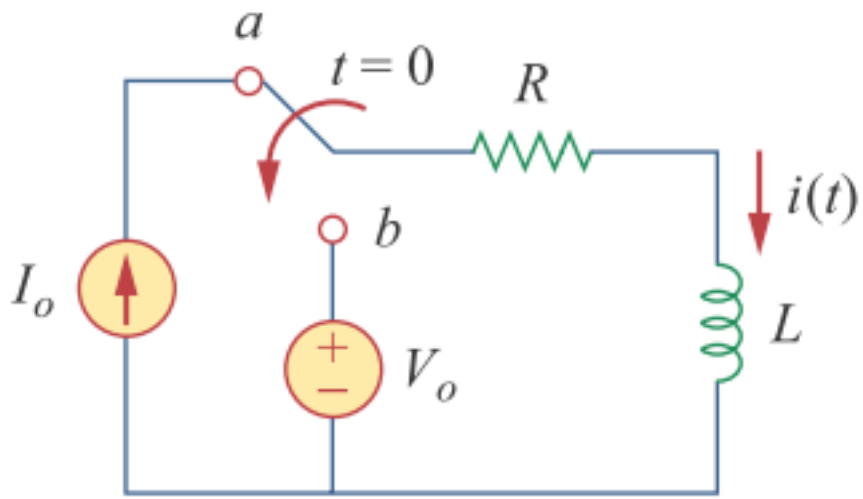
Ödev: $v_o(t)$'yi bulunuz.

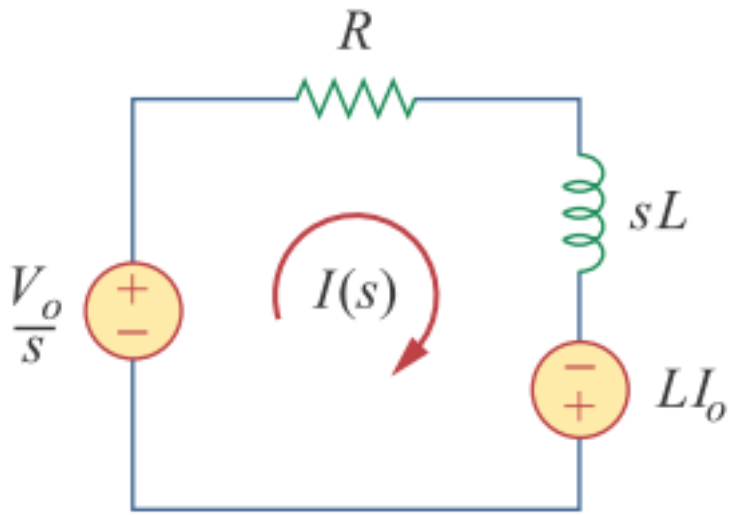


$$(24e^{-2t} - 4e^{-t/3})u(t) \text{ V}.$$

Devre Analizinde Laplace Dönüşümü

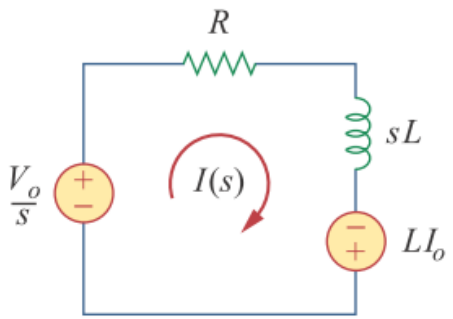
Soru: $t = 0$ anında anahtar a'dan b'ye çeviriliyor. $t > 0$ için $i(t)$'yi bulunuz.





Devre Analizinde Laplace Dönüşümü

Soru: $t = 0$ anında anahtar a'dan b'ye çeviriliyor. $t > 0$ için $i(t)$'yi bulunuz.



$$I(s)(R + sL) - LI_o - \frac{V_o}{s} = 0$$

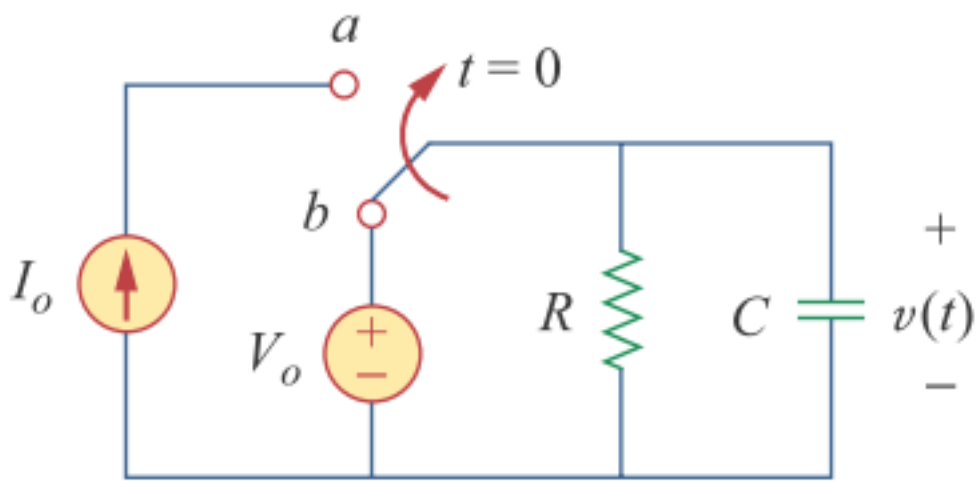
$$I(s) = \frac{LI_o}{R + sL} + \frac{V_o}{s(R + sL)} = \frac{I_o}{s + R/L} + \frac{V_o/L}{s(s + R/L)}$$

$$I(s) = \frac{I_o}{s + R/L} + \frac{V_o/R}{s} - \frac{V_o/R}{(s + R/L)}$$

$$i(t) = \left(I_o - \frac{V_o}{R}\right)e^{-t/\tau} + \frac{V_o}{R}, \qquad t \geq 0 \qquad \tau = \frac{L}{R}$$

Devre Analizinde Laplace Dönüşümü

Ödev: Anahtar b pozisyonunda uzusüre bekledikten sonra $t = 0$ anında a pozisyonuna getiriliyor. $t > 0$ için $v(t)$'yi bulunuz.

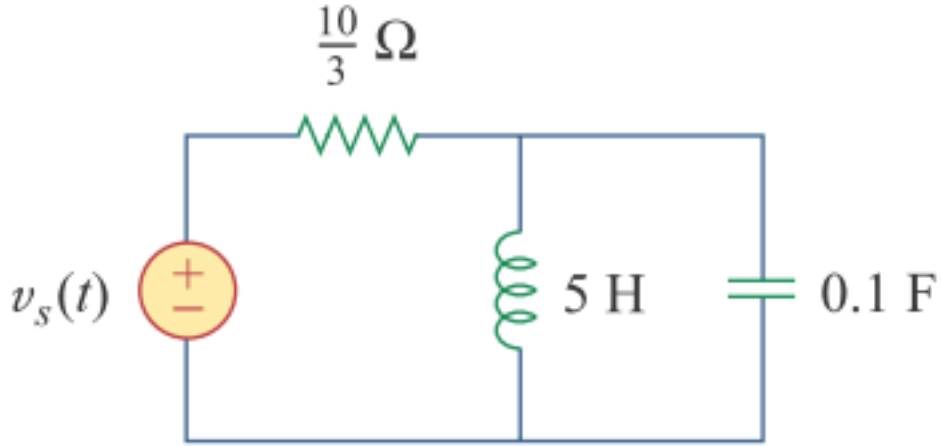


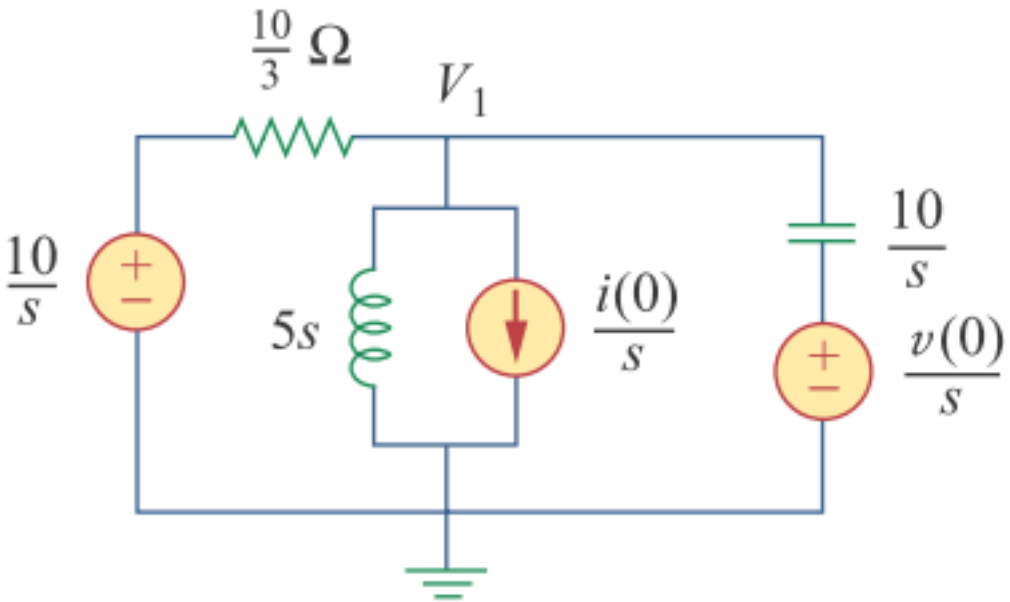
$$v(t) = (V_o - I_o R)e^{-t/\tau} + I_o R, t > 0, \text{ where } \tau = RC.$$

Devre Analizinde Laplace Dönüşümü

Soru: $t = 0$ anında bobinden -1A akım akıyor ve kapasitörde 5V voltaj var ise kapasitör üzerindeki voltajı düğüm gerilim yöntemi ile bulunuz.

$$v_s(t) = 10u(t)\ \text{V}$$





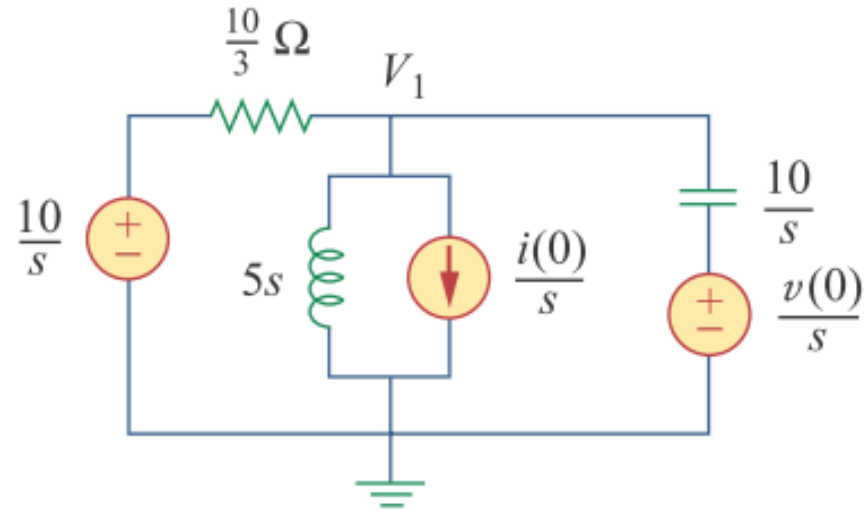
$$v(0) = 5\ \text{V}$$

$$i(0) = -1\ \text{A}$$

Devre Analizinde Laplace Dönüşümü

$$\frac{V_1 - 10/s}{10/3} + \frac{V_1 - 0}{5s} + \frac{i(0)}{s} + \frac{V_1 - [v(0)/s]}{1/(0.1s)} = 0$$

$$0.1\left(s + 3 + \frac{2}{s}\right) V_1 = \frac{3}{s} + \frac{1}{s} + 0.5$$



$$0.1\left(s + 3 + \frac{2}{s}\right) V_1 = \frac{3}{s} + \frac{1}{s} + 0.5$$

$$(s^2 + 3s + 2)\, V_1 = 40 + 5s$$

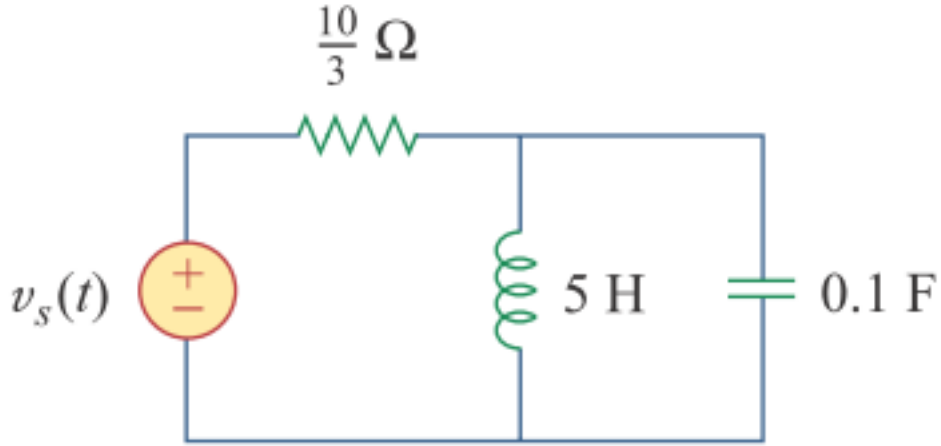
$$V_1 = \frac{40 + 5s}{(s + 1)(s + 2)} = \frac{35}{s + 1} - \frac{30}{s + 2}$$

$$v_1(t) = (35e^{-t} - 30e^{-2t})u(t)\ \text{V}$$

Devre Analizinde Laplace Dönüşümü

Ödev: Bir önceki soruda verilen devrede bobin üzerinden geçen akımı bulunuz.

$$v_s(t) = 10u(t)\ \text{V}$$



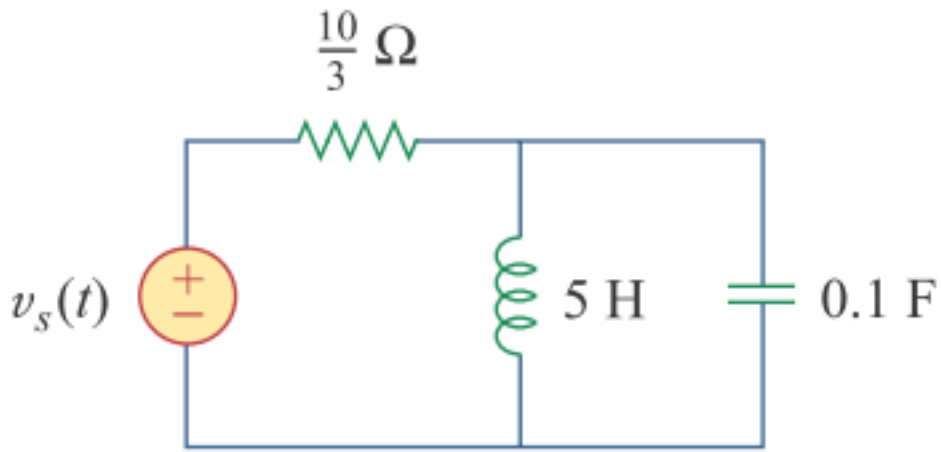
$$i(t) = (3 - 7e^{-t} + 3e^{-2t})u(t)\ \text{A}.$$

Devre Analizinde Laplace Dönüşümü

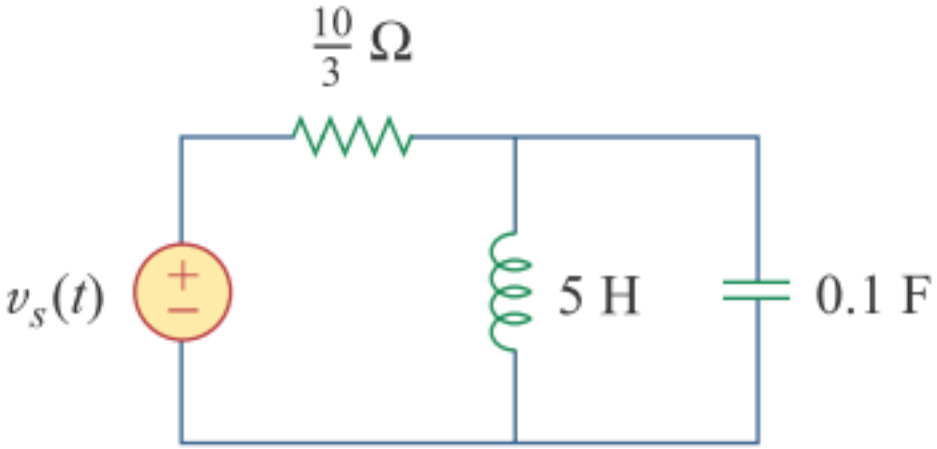
Soru: Bir önceki örnekte verilen devrede kapasitör üzerindeki voltajı bulmak için süperpozisyon yöntemini kullanınız.

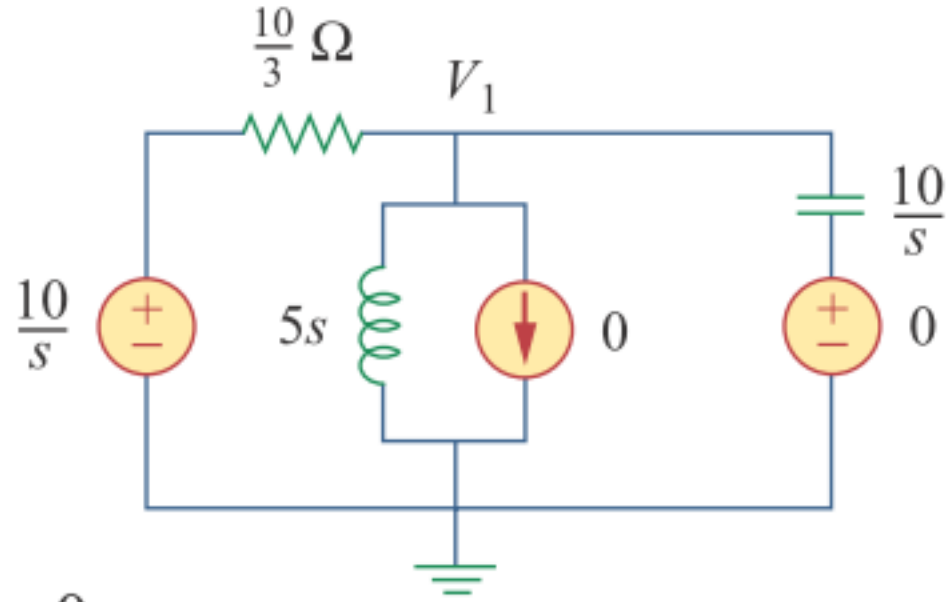
$t = 0$ anında bobinden -1A akım akıyor ve kapasitörde 5V voltaj var ise kapasitör üzerindeki voltajı bulunuz.

$$v_s(t) = 10u(t)\ \text{V}$$



Devre Analizinde Laplace Dönüşümü





$$\frac{V_1 - 10/s}{10/3} + \frac{V_1 - 0}{5s} + 0 + \frac{V_1 - 0}{1/(0.1s)} = 0$$

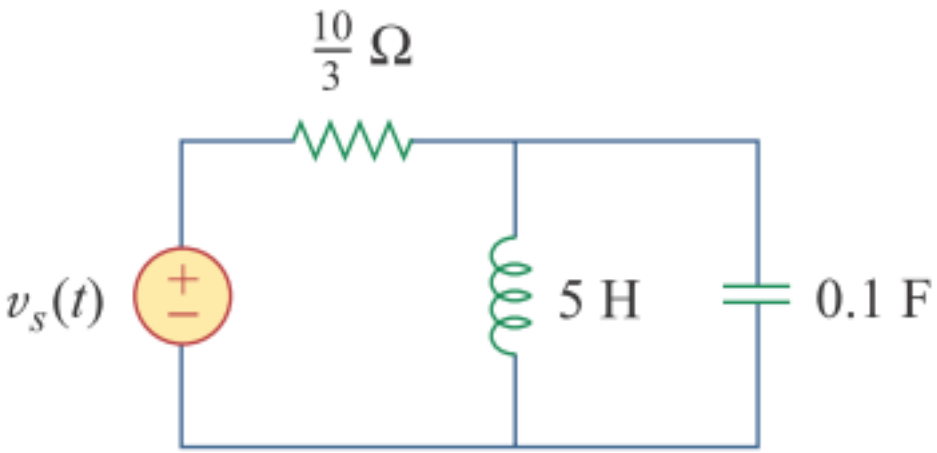
$$0.1\left(s + 3 + \frac{2}{s}\right)V_1 = \frac{3}{s}$$

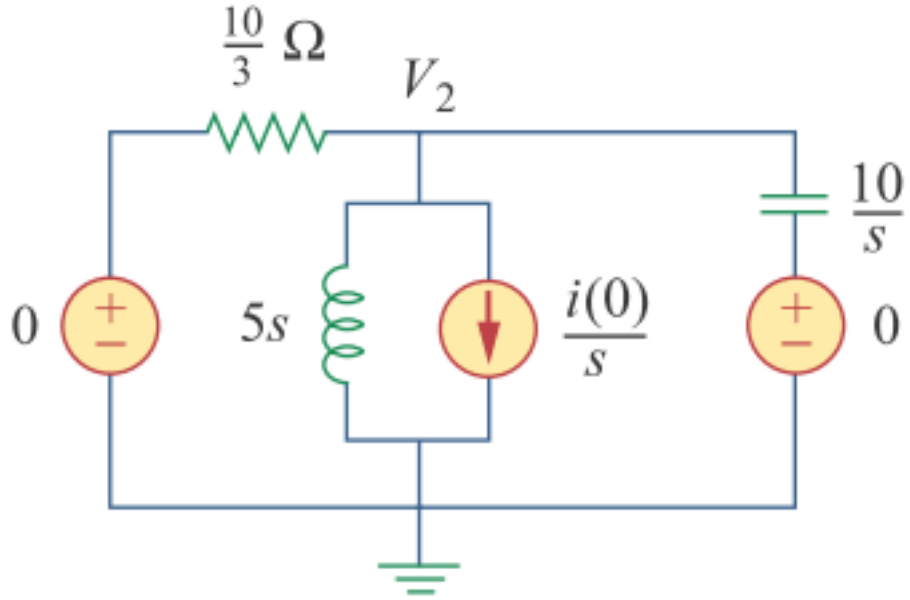
$$(s^2 + 3s + 2)V_1 = 30$$

$$V_1 = \frac{30}{(s+1)(s+2)} = \frac{30}{s+1} - \frac{30}{s+2}$$

$$v_1(t) = (30e^{-t} - 30e^{-2t})u(t)\ \text{V}$$

Devre Analizinde Laplace Dönüşümü





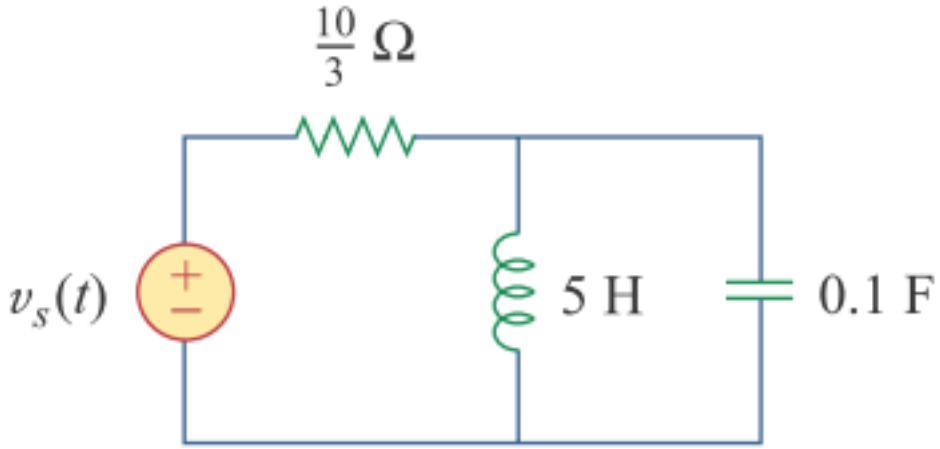
$$\frac{V_2 - 0}{10/3} + \frac{V_2 - 0}{5s} - \frac{1}{s} + \frac{V_2 - 0}{1/(0.1s)} = 0$$

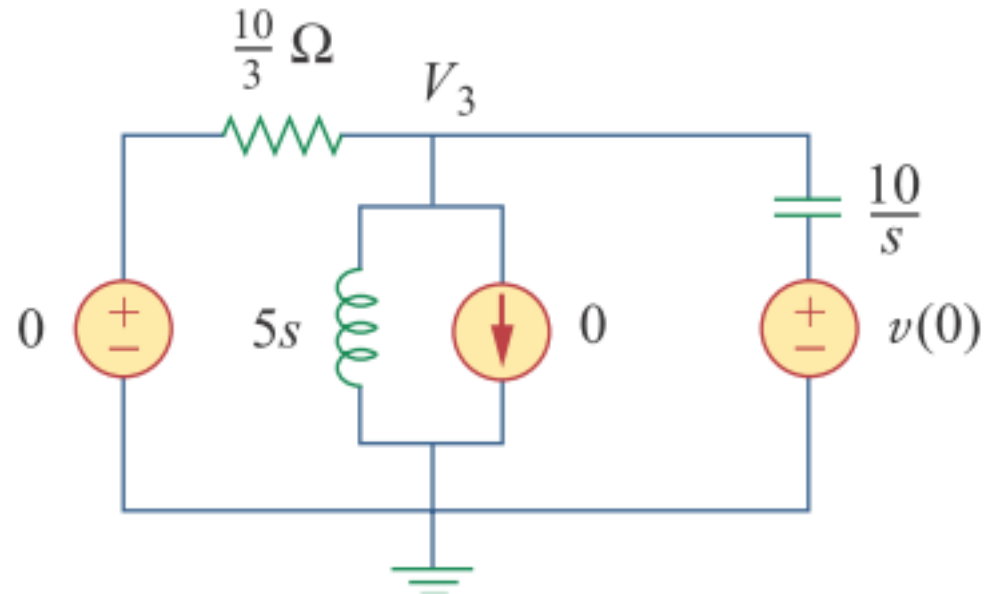
$$0.1\left(s + 3 + \frac{2}{s}\right) V_2 = \frac{1}{s}$$

$$v_2(t) = (10e^{-t} - 10e^{-2t})u(t)\ \text{V}$$

$$V_2 = \frac{10}{(s+1)(s+2)} = \frac{10}{s+1} - \frac{10}{s+2}$$

Devre Analizinde Laplace Dönüşümü





$$\frac{V_3 - 0}{10/3} + \frac{V_3 - 0}{5s} - 0 + \frac{V_3 - 5/s}{1/(0.1s)} = 0$$

$$0.1\left(s + 3 + \frac{2}{s}\right)V_3 = 0.5$$

$$V_3 = \frac{5s}{(s+1)(s+2)} = \frac{-5}{s+1} + \frac{10}{s+2}$$

$$v_3(t) = (-5e^{-t} + 10e^{-2t})u(t)\ \text{V}$$

Devre Analizinde Laplace Dönüşümü

$$v_1(t) = (30e^{-t} - 30e^{-2t})u(t)\ \text{V}$$

$$v_2(t) = (10e^{-t} - 10e^{-2t})u(t)\ \text{V}$$

$$v_3(t) = (-5e^{-t} + 10e^{-2t})u(t)\ \text{V}$$

$$\begin{aligned} v(t) &= v_1(t) + v_2(t) + v_3(t) \\ &= \{(30 + 10 - 5)e^{-t} + (-30 + 10 - 10)e^{-2t}\}u(t)\ \text{V} \end{aligned}$$

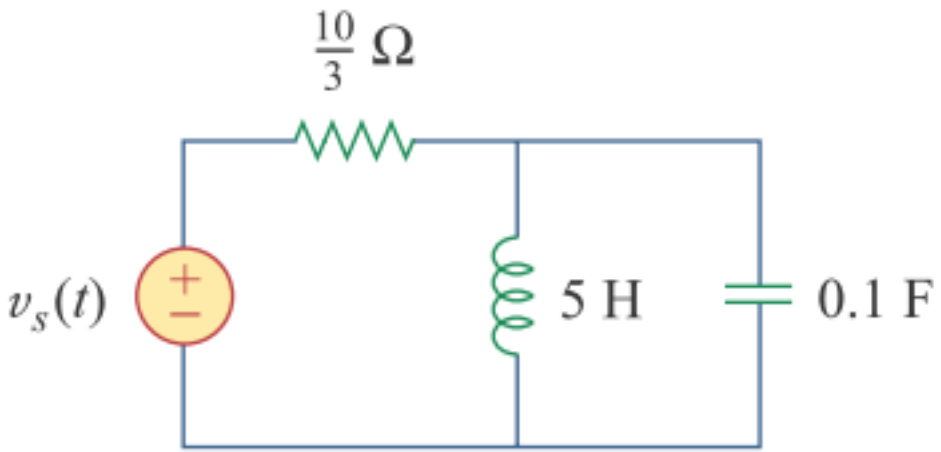
$$v(t) = (35e^{-t} - 30e^{-2t})u(t)\ \text{V}$$

Devre Analizinde Laplace Dönüşümü

Ödev: Bir önceki örnekte verilen devrede bobin üzerinden geçen akımı bulmak için süperpozisyon yöntemini kullanınız.

$t = 0$ anında bobinden -1A akım akıyor ve kapasitörde 5V voltaj var ise kapasitör üzerindeki voltajı bulunuz.

$$v_s(t) = 10u(t)\ \text{V}$$



$$i(t) = (3 - 7e^{-t} + 3e^{-2t})u(t)\ \text{A}.$$

Transfer Fonksiyonu

Transfer fonksiyonu sinyal işlemede önemli kavramlardan biridir. Bir sistemden geçen sinyalin nasıl davranacağını ifade eder. Transfer fonksiyonu s-uzayında çıkış sinyalinin giriş sinyaline oranı olarak tanımlanır. $X(s)$ giriş sinyali $Y(s)$ çıkış sinyali ise transfer fonksiyonu:

$$H(s) = \frac{Y(s)}{X(s)}$$

$$H(s) = \frac{V_o(s)}{V_i(s)}$$ Gerilim kazancı

$$H(s) = \frac{I_o(s)}{I_i(s)}$$ Akım kazancı

$$H(s) = \frac{V(s)}{I(s)}$$ Transfer Empedansı

$$H(s) = \frac{I(s)}{V(s)}$$ Transfer Admittansı

Transfer Fonksiyonu

Eğer sistemin input fonksiyonu ve transfer fonksiyonu biliniyorsa:

$$Y(s) = H(s)X(s)$$

Eğer giriş fonksiyonu darbe fonksiyonu ise:

$$x(t) = \delta(t) \qquad X(s) = 1$$

$$Y(s) = H(s) \qquad y(t) = h(t) \qquad h(t) = \mathcal{L}^{-1}[H(s)]$$

Birim darbe cevabı

Transfer Fonksiyonu

Soru: Bir lineer sistemin input ve outputu aşağıda verilmiştir. Transfer fonksiyonunu ve darbe cevabını bulunuz.

$$y(t) = 10e^{-t}\cos 4t\, u(t) \qquad x(t) = e^{-t}u(t)$$

$$Y(s) = \frac{10(s+1)}{(s+1)^2 + 4^2} \qquad X(s) = \frac{1}{s+1}$$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{10(s+1)^2}{(s+1)^2 + 16} = \frac{10(s^2 + 2s + 1)}{s^2 + 2s + 17}$$

$$H(s) = 10 - 40\frac{4}{(s+1)^2 + 4^2} \qquad h(t) = 10\delta(t) - 40e^{-t}\sin 4t\, u(t)$$

Transfer Fonksiyonu

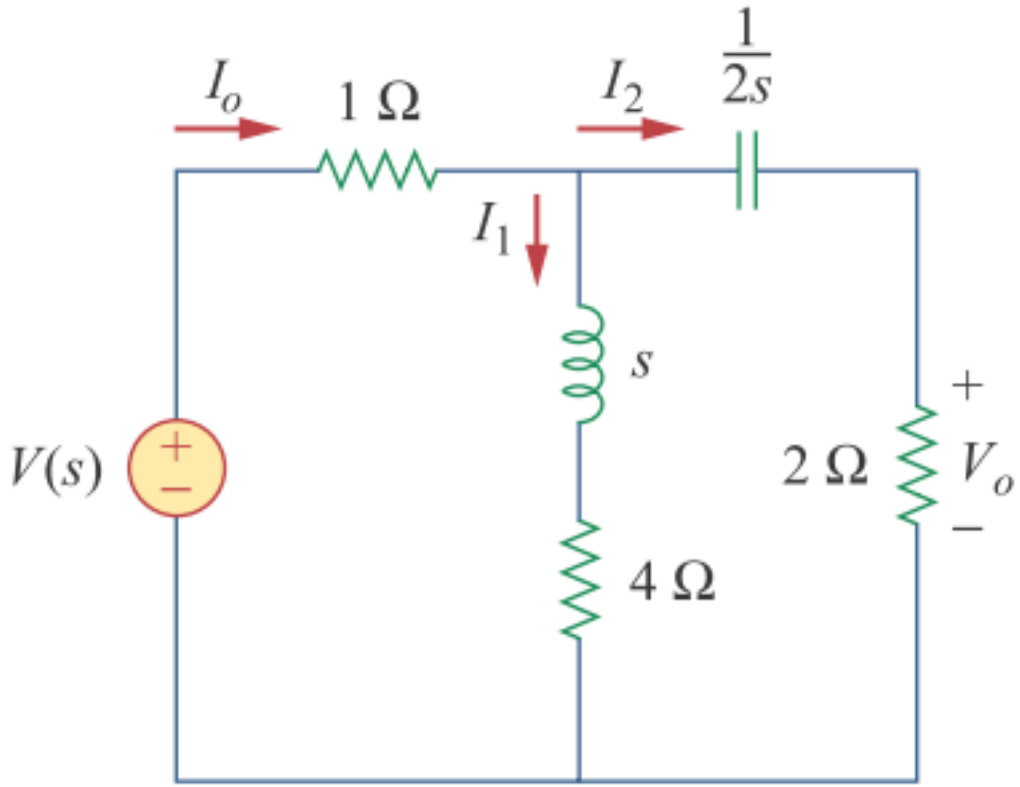
Ödev:Transfer fonksiyonu verilen bir lineer sisteme $5e^{-3t}u(t)$ input sinyali uygulanıyorsa $y(t)$'yi bulunuz.

$$H(s) = \frac{2s}{s+6}$$

$$-10e^{-3t} + 20e^{-6t}, t \geq 0.$$

Transfer Fonksiyonu

Soru: Verilen devrede $H(s) = V_o(s)/I_o(s)$ transfer fonksiyonunu bulunuz.



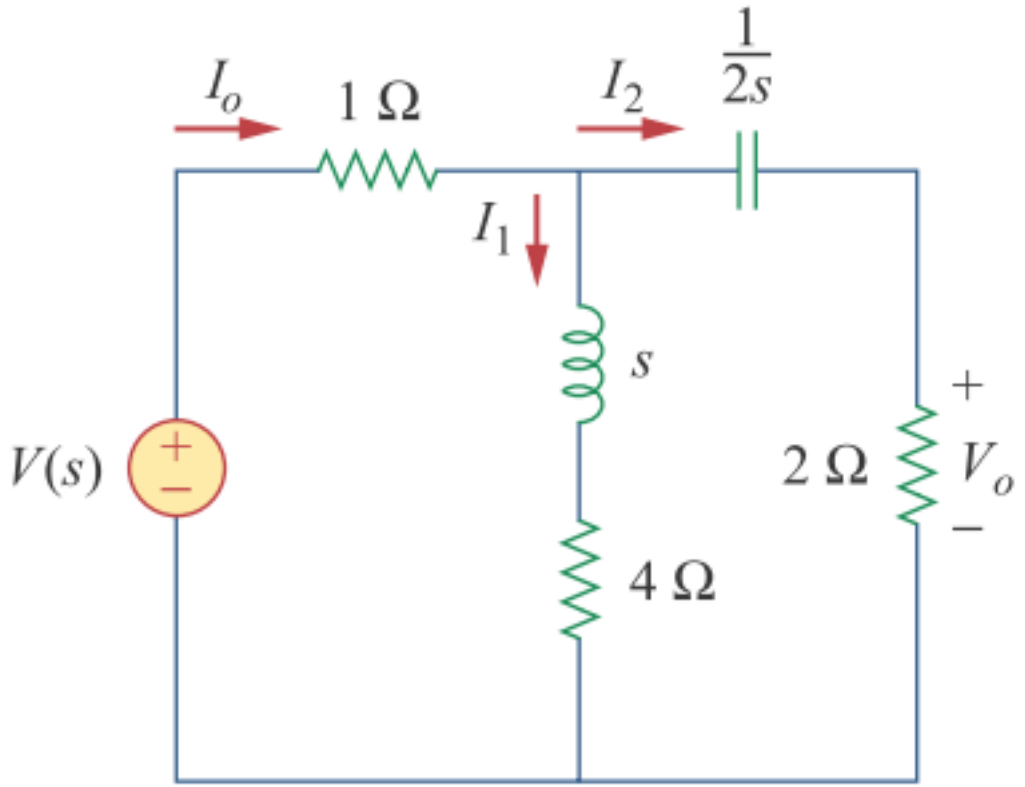
$$I_2 = \frac{(s+4)I_o}{s+4+2+1/2s}$$

$$V_o = 2I_2 = \frac{2(s+4)I_o}{s+6+1/2s}$$

$$H(s) = \frac{V_o(s)}{I_o(s)} = \frac{4s(s+4)}{2s^2+12s+1}$$

Transfer Fonksiyonu

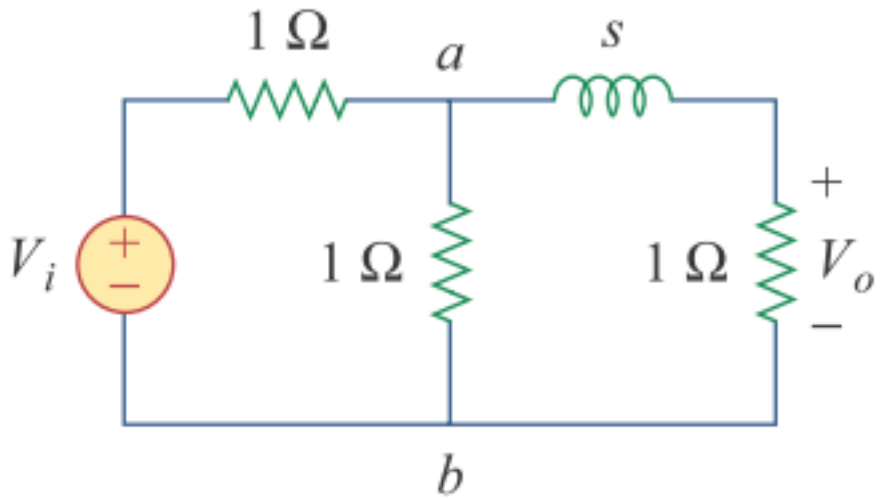
Ödev: Verilen devrede $H(s) = I_1(s)/I_0(s)$ transfer fonksiyonunu bulunuz.



$$\frac{4s + 1}{2s^2 + 12s + 1}.$$

Transfer Fonksiyonu

Soru: Verilen devrede $H(s) = V_o/V_i$ transfer fonksiyonunu bulunuz. Darbe cevabını, $v_i = u(t)$ ve $v_i = 8\cos 2t$ olduğunda sistemin cevabını bulunuz.



$$V_o = \frac{1}{s+1} V_{ab}$$

$$V_{ab} = \frac{1 \| (s+1)}{1 + 1 \| (s+1)} V_i = \frac{(s+1)/(s+2)}{1 + (s+1)/(s+2)} V_i$$

$$V_{ab} = \frac{s+1}{2s+3} V_i \qquad V_o = \frac{V_i}{2s+3} \qquad H(s) = \frac{V_o}{V_i} = \frac{1}{2s+3}$$

Transfer Fonksiyonu

$$H(s) = \frac{V_o}{V_i} = \frac{1}{2s + 3} \qquad H(s) = \frac{1}{2}\frac{1}{s + \frac{3}{2}}$$

$$h(t) = \frac{1}{2}e^{-3t/2}u(t)$$ darbe cevabı

$$v_i(t) = u(t),\ V_i(s) = 1/s$$

$$V_o(s) = H(s)V_i(s) = \frac{1}{2s(s + \frac{3}{2})} = \frac{A}{s} + \frac{B}{s + \frac{3}{2}}$$

Transfer Fonksiyonu

$$V_o(s) = H(s)V_i(s) = \frac{1}{2s(s+\frac{3}{2})} = \frac{A}{s} + \frac{B}{s+\frac{3}{2}}$$

$$A = sV_o(s)\big|_{s=0} = \frac{1}{2(s+\frac{3}{2})}\bigg|_{s=0} = \frac{1}{3}$$

$$B = \left(s+\frac{3}{2}\right)V_o(s)\bigg|_{s=-3/2} = \frac{1}{2s}\bigg|_{s=-3/2} = -\frac{1}{3}$$

$$V_o(s) = \frac{1}{3}\left(\frac{1}{s} - \frac{1}{s+\frac{3}{2}}\right) \qquad v_o(t) = \frac{1}{3}(1 - e^{-3t/2})u(t)\ \text{V}$$

Transfer Fonksiyonu

$$H(s) = \frac{V_o}{V_i} = \frac{1}{2s + 3} \qquad v_i(t) = 8\cos 2t, \qquad V_i(s) = \frac{8s}{s^2 + 4},$$

$$V_o(s) = H(s)V_i(s) = \frac{4s}{(s + \frac{3}{2})(s^2 + 4)}$$

$$= \frac{A}{s + \frac{3}{2}} + \frac{Bs + C}{s^2 + 4}$$

$$A = \left(s + \frac{3}{2}\right) V_o(s)\Big|_{s=-3/2} = \frac{4s}{s^2 + 4}\Big|_{s=-3/2} = -\frac{24}{25}$$

Her iki tarafı

$(s + 3/2)(s^2 + 4)$ $\quad 4s = A(s^2 + 4) + B\left(s^2 + \frac{3}{2}s\right) + C\left(s + \frac{3}{2}\right)$

çarpalım.

Transfer Fonksiyonu

$$4s = A(s^2 + 4) + B\left(s^2 + \frac{3}{2}s\right) + C\left(s + \frac{3}{2}\right) \qquad A = -\frac{24}{25}$$

s=0 iken $\quad 0 = 4A + \frac{3}{2}C \quad \Rightarrow \quad C = -\frac{8}{3}A \qquad C = 64/25$

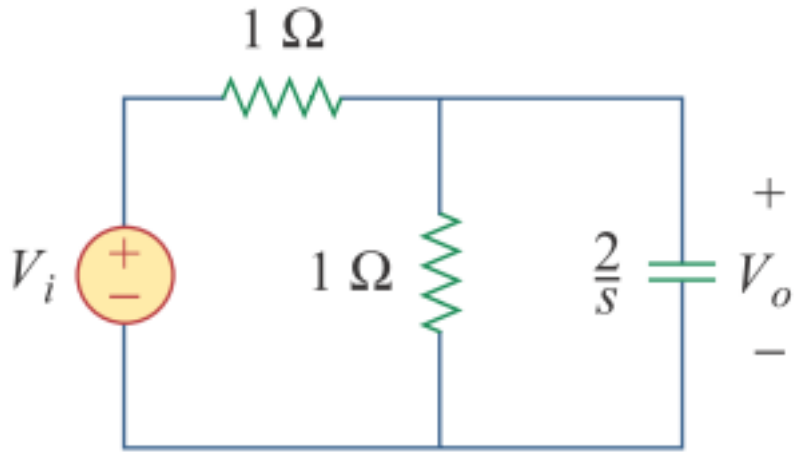
s=1 iken $\quad 4 = 5A + (5/2)B + (5/2)C \qquad B = 24/25$

$$V_o(s) = \frac{-\frac{24}{25}}{s + \frac{3}{2}} + \frac{24}{25}\frac{s}{s^2 + 4} + \frac{32}{25}\frac{2}{s^2 + 4}$$

$$v_o(t) = \frac{24}{25}\left(-e^{-3t/2} + \cos 2t + \frac{4}{3}\sin 2t\right)u(t)\ \text{V}$$

Transfer Fonksiyonu

Ödev: Verilen devrede $H(s) = V_o/V_i$ transfer fonksiyonunu bulunuz. Darbe cevabını, $v_i = u(t)$ ve $v_i = 8\cos 2t$ olduğunda sistemin cevabını bulunuz.



$$2/(s+4)$$

$$2e^{-4t}u(t)$$

$$\tfrac{1}{2}(1-e^{-4t})u(t)\ \text{V}$$

$$3.2(-e^{-4t}+\cos 2t+\tfrac{1}{2}\sin 2t)\,u(t)\ \text{V}$$